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Turbulent mass flux measurements using a laser/hot-wire technique

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Abstract—A laser-Doppler anemometer and a hot-wire type concentration probe are used to measure the velocity and species concentration fields simultaneously in an axisymmetric sudden-expansion flow. The presence of the probe has a negligible effect on the Doppler signals. Furthermore, the presence of small liquid droplets in the flow does not affect the hot-wire measurements. Resulting measurements of axial velocity and species concentration are compared with corresponding data obtained by applying the LDA and hot-wire probe separately to measure the flow properties. Excellent agreement is obtained. Therefore, the measured mass flux is valid and reliable. The technique is used to examine the behavior of the turbulent mass flux in an axisymmetric sudden-expansion flow.

1. INTRODUCTION

THERE are many practical examples of turbulent flow with significant density variations and fluctuations. Some of these are: the wakes behind bodies in supersonic or hypersonic flights, flows inside combustors, boundary layers on bodies in supersonic or hypersonic flights, geophysical flows and binary mixing of fluids. With the advent of high-speed computers, sophisticated computational schemes are formulated to calculate these flows with the hope that practical procedures can be developed to design missiles, propulsion devices, combustors, etc. These schemes need more than just mean flow knowledge to verify and demand accurate measurements of fluctuating quantities and turbulent fluxes in variable-density flows.

Hot-wire probes have been developed to measure the instantaneous species concentration [1] and to resolve both velocity and species concentration simultaneously [2]. With the development of these probes, it is possible to study density effects in turbulent mixing layers [3], in turbulent jets [4], in turbulent boundary layers with injection [5] and in helium–air mixtures [6]. In the mixing layer study [3], the concentration probe of ref. [1] is used to measure the instantaneous density variation across the mixing layer. From these data, large-scale motions in the mixing layer are identified and density fluctuations are determined. In other studies [4–6], the hot-wire probe of ref. [2] is used to resolve simultaneously the velocity and species concentration fields. Therefore, for the first time turbulent mass fluxes are measured in turbulent, variable-density flows. Unfortunately, these studies are carried out to demonstrate the viability, validity and accuracy of the hot-wire technique and the various flows have not been studied in detail.

Optical techniques have also been developed to measure species concentration in a turbulent, variable-density flow. These consist of a fiber optic probe [7], Raman spectrometer [8] and Rayleigh scattering [9–11] techniques. The fiber optic probe detects species concentration because the passage of eddies containing the species causes light to attenuate in the probe gap. It is possible to use the probe together with hot-wire probes to measure velocity, concentration and temperature simultaneously, thus allowing their respective turbulent fluxes to be determined [7]. This technique has been used to study plane mixing layers with and without reactions and leads to significant insight into the mixing behavior of variable-density flows. The Raman spectrometer of ref. [8] is tuned to detect the Stokes Raman band of methane and the resulting radiation is detected by a suitable photomultiplier and processed for analysis. This technique has been used by other investigators to measure species concentration in non-reacting as well as reacting flows [12–14]. Its simultaneous use with the laser-Doppler anemometer (LDA) technique has also been attempted [13, 14]. On the other hand, the Rayleigh scattering technique requires the mixture scattering cross-section to be known. Thus, its application is limited to those flows where this information is available. Again, the technique can be used simultaneously with LDA [15, 16]. Therefore, it can be used to determine turbulent fluxes in variable-density flows with and without chemical reactions [11, 13, 15, 16].

This paper presents the development and verification of a laser/hot-wire technique for the simultaneous measurement of velocity and species concentration. The technique is based on the use of a laser-Doppler anemometer and a modified version of the hot-wire concentration probe of Brown and Rebollo [1]. The modified concentration probe has been used to measure helium concentration in a variety of flows [17–19]. These include complex turbulent flows with and without swirl [17, 18], flows with recir-

NOMENCLATURE

$\tilde{c}$	instantaneous helium volume concentration
c	fluctuating part of $\tilde{c}$
c'	r.m.s. of c
C	mean of $\tilde{c}$
d_1	diameter of inlet nozzle
d_2	diameter of downstream tube
$E(f)$	one-dimensional energy spectrum
f	frequency [s^{-1}]
H	step height, $(d_2-d_1)/2$
k	wave number, $2\pi f/U$
r	radial coordinate measured from tube center
R	radius of downstream tube, $d_2/2$
$\tilde{u}$	instantaneous axial velocity
u	fluctuating part of $\tilde{u}$
u'	r.m.s. of u
U	mean of $\tilde{u}$
$(UC)_{av}$	average UC value across tube, $2/R^2\int_0^R UCr\,dr$
$\tilde{v}$	instantaneous radial velocity
v	fluctuating part of $\tilde{v}$
$\tilde{w}$	instantaneous circumferential velocity
w	fluctuating part of $\tilde{w}$
x	axial coordinate measured from entrance to sudden expansion
x_L	reattachment length.

Greek symbols

α	density ratio parameter, (ρ_h/ρ_a-1)
η	Kolmogorov length scale
$\tilde{\theta}$	instantaneous mixture mass fraction
θ	fluctuating part of $\tilde{\theta}$
θ'	r.m.s. of θ
Θ	mean of $\tilde{\theta}$
$\tilde{\rho}$	instantaneous density of mixture
ρ	fluctuating part of $\tilde{\rho}$
ρ'	r.m.s. of ρ
$\bar{\rho}$	mean of $\tilde{\rho}$
τ	Kolmogorov time scale
ψ	stream function.

Subscripts

a	air
c	helium volume concentration
℄	centerline values
h	helium
i	ith component of velocity
j	central air jet condition
u	axial velocity
uc	uc correlation
0	annular helium/air jet condition.

culation regions and rotating flows [19]. The concentration measurements are shown to be in good agreement with calculations [20] and are quite independent of fluid rotation in the flow. Therefore, the probe gives reliable concentration measurements for a wide variety of complex turbulent flows. Since it is much less complicated than the laser scattering techniques [8–16] and quite a bit cheaper, it is most suitable for species concentration measurements in all kinds of turbulent flows. If the technique is used in conjunction with LDA, a convenient, reliable and much less costly diagnostic technique is available for the study of turbulent mass fluxes in variable-density flows.

The laser/hot-wire technique is described in Section 2. This is followed by a brief description of the test rig where the technique is used to measure flow properties. Validation of the technique is discussed in Section 4, while the measurements of turbulent mass flux are presented in Section 5. Finally, the study is summarized in Section 6.

2. LASER/HOT-WIRE MEASUREMENT TECHNIQUE

The diagnostic instruments used in this technique consist of an LDA and a hot-wire anemometer. A standard DISA Model 55X laser-Doppler anemometer (LDA) equipped with a DISA Model 55N10–1 frequency shifter is used to measure $\tilde{u}$ and $\tilde{w}$. The coherent light source is provided by a 15 mW helium–neon laser (wavelength = 632.8 nm) and the light beam is split using a Bragg cell. One of the beams is shifted 40 MHz to enable the detection of reversed flow and high turbulence in the flow field. The focusing lens has a focal length of 80 mm. This allows the beam separation to be chosen at 25.2 mm and the beam intersection angle is 17.9°. The resultant ellipsoidal sampling volume has dimensions of 0.26 mm length and 0.04 mm width. A DISA Model 59L 90A counter processor is used to amplify and filter the signal fed by the frequency shifter. The counter processor is also equipped with a D/A converter. Therefore, either a digital or an analog LDA signal is available from the counter processor. Artificial seeding provided by a DISA Model 55L18 seeding generator is used to improve the LDA signal quality and frequency resolution. Liquid (50% water and 50% glycerine) droplets centered around 1 μm in size are generated and carried into the test section by an air stream. The droplet concentration is regulated by varying the air inlet pressure to the seeding generator. Details of the LDA system are provided by So *et al.* [21].

The hot-wire concentration probe of Ahmed and So [17] is used to measure the helium volume concentration in the sudden-expansion flow. Calibration procedure is given in ref. [17]. Associated hot-wire instruments used with the probe consist of a DISA

Model 56C01 constant temperature anemometer (CTA) and a Model 56C16 general purpose CTA bridge. The hot-wire concentration probe has an opening 0.13 mm in diameter and the hot-wire sensor is located 12.7 mm downstream of the probe tip. If the probe is operated with choked flow at the tip as suggested by Brown and Rebollo [1], the sensor is sensitive to helium volume concentration only when the upstream stagnation condition and the probe back-pressure are constant. So and co-workers [18, 19] found that the probe sensor is not sensitive to any upstream velocity variations even when fluid rotation is present in the flow field. The probe capture volume is estimated to have a diameter of ~ 0.35 mm and the time delay from probe tip to sensor location is determined to be $\sim 7\,\mu s$ using one-dimensional gas dynamic analysis. Although the sensor is situated at 12.7 mm downstream of the probe tip, the measurement is considered to be at the location of the probe tip because of the small time delay. The time delay can be considered small because it is indeed small compared to the estimated Kolmogorov time scale (τ) in the flow. Based on the test conditions selected, $\tau \simeq 50\,\mu s$. This extremely small time delay, therefore, represents an advantage in the development of the LDA/hot-wire technique to measure turbulent mass fluxes. Further details about the probe and the CTA system are available in refs. [17–19].

The flux measurement technique consists of using the LDA and hot-wire concentration probe simultaneously to measure the velocity and species concentration fields. In order to satisfy the requirement that the velocity and species concentration fields are sampled at the same point and at the same time, the probe should be placed directly above or slightly ahead of the LDA beam volume. However, this would disturb the transmitted light beam signal. Therefore, the probe tip is displaced slightly behind and downstream of the LDA beam volume. A simple experiment is carried out to determine the optimum displacement of the probe from the LDA beam volume. The LDA, with and without the presence of the hot-wire concentration probe, is set up to measure the equilibrium properties of a round free air jet. When the probe is present, the displacement of the probe tip from the LDA beam volume is slowly varied until no change is observed in the LDA signal. Results of the measurements of U and u' are plotted in Fig. 1, and show that the presence of the probe does not adversely affect the LDA measurements. Furthermore, the measurements agree with the theoretical calculations of ref. [22]. The resultant displacement is found to be 0.4 mm behind the major axis of the LDA probe volume. Thus, the dimensions of the combined LDA and concentration probe volume are approximately 0.5 mm long by 0.5 mm wide. The Kolmogorov length scale, η, for the present experiment is estimated to be about 0.1 mm. Even though the dimensions of the present measurement volume are quite a bit larger than η, they are comparable to those of Dibble *et al.* [14] (combined LDA/Raman scattering), Schefer and Dibble [15] and Driscoll *et al.* [16] (combined LDA/Rayleigh scattering) used in similar flow situations. Therefore, the present technique is subject to the same kind of inaccuracy in spatial resolution as the techniques of refs. [14–16].

The analog LDA and hot-wire signals are digitized using a Metrabyte-16 A/D converter. Resolution is 12

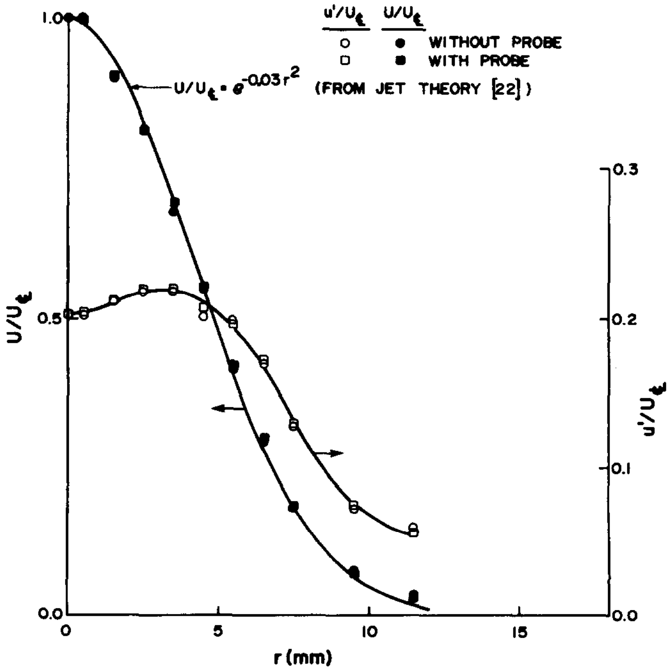
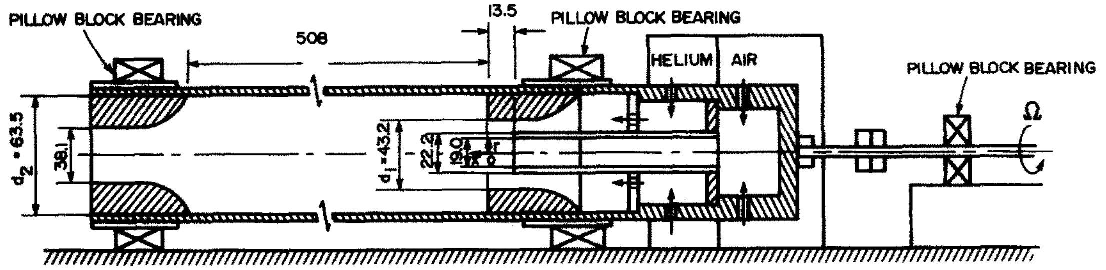


Fig. 1. LDA measurements of a round free air jet with and without the presence of a concentration probe.



NOTE: ALL DIMENSIONS IN mm

FIG. 2. Schematic of test section.

bit and the total through-put rate for 8 channels is 35 kHz. The digitized signals are analyzed using a Zenith ZF-158 personal computer equipped with a 45 Mb Winchester drive and a 642 kb RAM. If on-line analysis is not required, the data is recorded on disc for transfer to a mainframe computer. There, the signals can be analyzed in detail. For the present experiment, a suitable A/D conversion rate is found to be 2000 samples per second because the spectral energies of $\overline{u^2}$ and $\overline{c^2}$ are more than 2 decade down at a frequency of 1 kHz (see spectra results in Section 5). A simple experiment is performed to determine the suitable record length for the calculation of the statistics of the signals. The result shows that the calculated mean and r.m.s. quantities are essentially independent of the data record length beyond a length of 5 s. Therefore, data record lengths of 5 s are used to calculate the statistics and power spectra.

The volume concentration, $\tilde{c}$, measured by the probe is related to the mass fraction, $\tilde{\theta}$, by

$$\tilde{\theta} = \frac{(1+\alpha)\tilde{c}}{1+\alpha\tilde{c}}. \tag{1}$$

Therefore, the quantities $\overline{u_i\theta}$ can be related to $\overline{u_ic}$. If $\overline{\theta c}$ is assumed to be small compared to ΘC then

$$\Theta = \frac{(1+\alpha)C}{1+\alpha C}. \tag{2}$$

Similarly, if $\alpha\overline{u_i\theta c}$ is small compared to $(1+\alpha)\overline{u_ic}$ and is neglected, then

$$\overline{u_i\theta} = \frac{(1+\alpha)\overline{u_ic}}{(1+\alpha C)^2}. \tag{3}$$

On the other hand, $\tilde{\rho}$ is related to $\tilde{c}$ by

$$\tilde{\rho} = \rho_a(1-\tilde{c})+\rho_h\tilde{c}. \tag{4}$$

Therefore

$$\bar{\rho} = \rho_a(1+\alpha C) \tag{5}$$

$$\overline{\rho u_i} = \alpha\rho_a(\overline{u_ic}) \tag{6}$$

and no approximations are necessary. Once $\overline{u_ic}$ are measured, the corresponding $\overline{u_i\theta}$ and $\overline{\rho u_i}$ are calculated from the above equations.

3. EXPERIMENTAL SET-UP

The test facility is a simple rotating rig with a sudden expansion geometry. A schematic is shown in Fig. 2. It consists of a Plexiglas tube of $d_2 = 63.5$ mm and 76.2 cm long. The tube is supported by two pillow block bearings, one at each end of the tube. Both ends of the tube are fitted with well-contoured nozzles made of aluminum. They are arranged as shown in Fig. 2 and are exactly $8d_2$ away from each other. A hollow aluminum cylinder consisting of two compartments and an inner hollow tube is installed against the inlet nozzle. The dimensions are as indicated in Fig. 2. A steel shaft is attached to the closed-end of the aluminum cylinder and, in turn, is connected to the motor drive shaft through a Gerbing coupling. The drive shaft is also supported by two pillow block bearings. Different gases can be admitted to the two compartments through equally-spaced holes drilled around the circumference of the aluminum cylinder. Therefore, the flow is from the shaft-end to the open-end as shown in Fig. 2. Two settling chambers are used to enclose the open-end (not shown in Fig. 2) and shaft-end of the assembly. Air and a helium/air mixture are supplied to the inlet chamber as shown and the flow is pulled through by a blower attached to the chamber at the outlet.

The same rig had been used by So *et al.* [19] to study the effects of rotation on the mixing of an inner air stream with an annular helium/air stream in an axisymmetric sudden-expansion flow. In that investigation, LDA was first used to study the flow field. Then the concentration probe was used to measure the helium volume concentration. Therefore, the velocity and scalar fields were measured separately. Consequently, the LDA measurements were not influenced by the presence of the concentration probe and the helium volume concentration measurements were not affected by the presence of liquid droplets in the flow. Their studies were carried out for two rotational speeds; one stationary and one with a rotational speed of $\Omega = 88\ \mathrm{rad\,s^{-1}}$. The present experiment is again carried out with an inner air stream and an annular helium/air stream. Only the stationary experiment is carried out with the same inlet conditions. These conditions, measured at the exit plane of the inner tube or $x = -13.5$ mm (see Fig. 2), are: $C_0 = 0.54$, $U_0 = 8\ \mathrm{m\,s^{-1}}$ and $U_j = 24\ \mathrm{m\,s^{-1}}$. The velocities are found to be quite constant across the test section. However, the velocity ratio U_j/U_0 is found to vary slightly during the course of the experiment. Its value is determined to be 3 ± 0.05. Also, C_0 varies between

0.54 and 0.55. The density ratio, ρ_0/ρ_j, is determined to be approximately 0.54. Therefore, the inner air jet is twice as heavy as the annular helium/air jet.

Simultaneous LDA and concentration measurements are carried out at different x locations ranging from $x/H = 0.54$ to 30.6. Most of these locations are selected to be the same as those chosen by So *et al.* [19], so that the present measurements can be compared with the results of ref. [19]. The LDA is set up to measure $\tilde{u}$ only. Therefore, from the simultaneous measurements of $\tilde{u}$ and $\tilde{c}$ the statistical quantities, U, u', C, c' and $\overline{uc}$ are calculated. The data U, u', C and c' are compared with the measurements of ref. [19] for accuracy assessment.

A special two-dimensional, manual traversing mechanism is built for the present experiment. Both the LDA and the concentration probe system are mounted on this traversing device. The L-shape concentration probe is introduced into the test section through the side of the tube and is set up at a distance 0.4 mm behind and 0.2 mm above the LDA beam volume. Once this is properly set up, the probe is fixed and the radial and axial traverse is controlled by the manual two-dimensional traversing mechanism which is accurate to 0.2 mm in the radial direction and to 0.5 mm in the axial direction.

4. VALIDATION OF LASER/HOT-WIRE TECHNIQUE

With the exception of the location at $x = -13.5$ mm, all measurements at other x locations are obtained simultaneously. The simultaneous measurements of $\tilde{u}$ and $\tilde{c}$ are first analyzed for their velocity and helium volume concentration behavior downstream of the sudden expansion. Since So *et al.* [19] have also studied this flow by using the LDA and concentration probe separately, a comparison of the present results with theirs will serve to validate the laser/hot-wire technique and its accuracy. Even though in both cases, measurements at ten different x locations are available, the present comparison is shown with seven locations only. This does not mean that the measurements in the other locations do not compare well. The intent here is to give a clear comparison of the measured values.

Velocity results (U and u') are compared in Fig. 3, while concentration results (C and c') are shown in Fig. 4. The centerline behaviors of U, u', C and c' are plotted in Fig. 5. The x locations chosen in Figs. 3 and 4 vary from $x/H = 0.5$ to 24.6 and cover the recirculation region behind the sudden expansion and the recovery region downstream of the reattachment point. It can be seen that the agreement between the present measurements and those of ref. [19] is excellent in all x locations measured. There are isolated discrepancies, such as the C profiles at $x/H = 8.6$ where significant disagreement occurs in the near-wall region. In general, all the differences observed in the measurements are within the error bounds of the concentration probe and LDA techniques.

A further validation of the present technique is obtained by comparing the measured reattachment length in this flow with that determined in ref. [19]. As before, the measured U and C are used to calculate the stream function ψ, given by

$$\psi = \int_0^r \rho_a(1+\alpha C)Ur\,\mathrm{d}r. \qquad (7)$$

The $\psi = 0$ curve is extrapolated to the wall to give x_L, the reattachment length. Thus determined, x_L/H is found to be 9.5 ± 1. This compares with a value of 9.7 given in ref. [19]. Therefore, the present measurements of $\tilde{u}$ and $\tilde{c}$ are shown to be quite accurate. This shows that the presence of the concentration probe in the

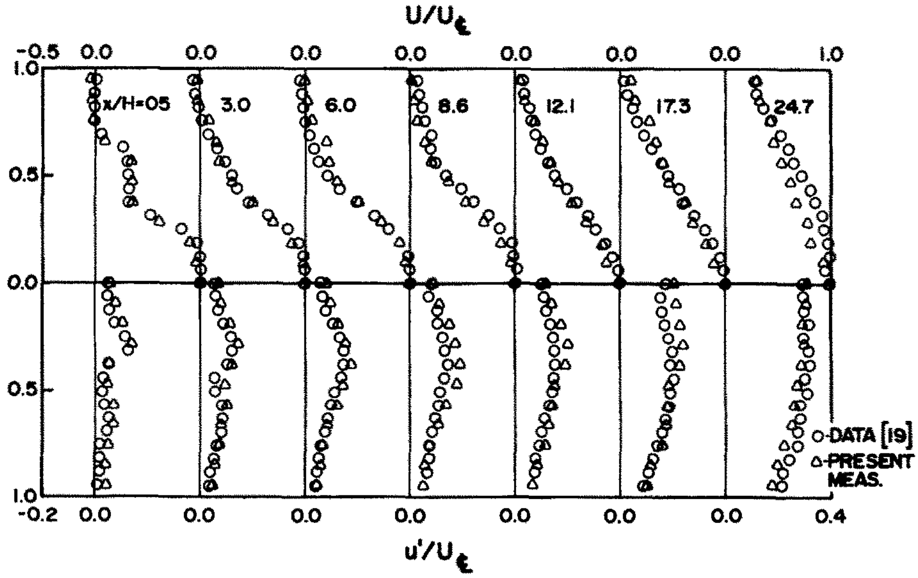


FIG. 3. Comparison of measured U and u' with measurements of ref. [19].

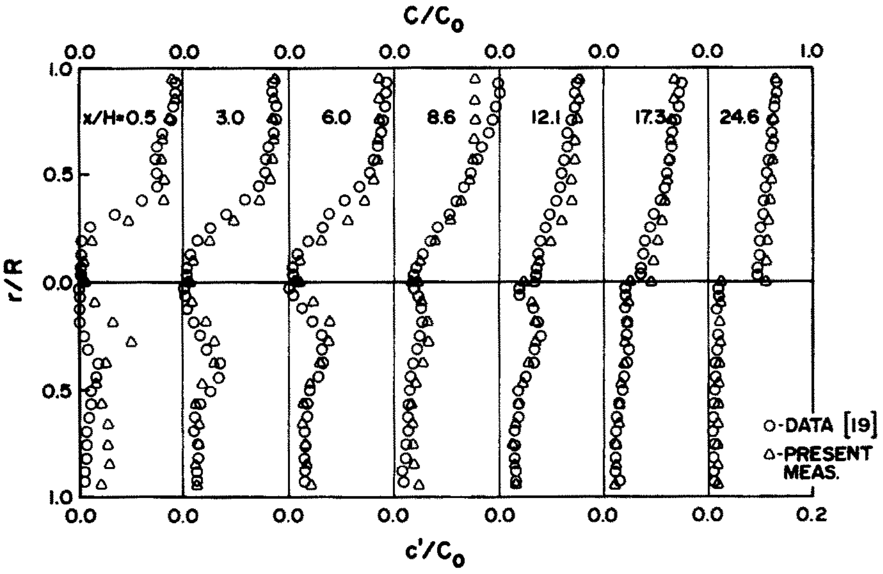


FIG. 4. Comparison of measured C and c' with measurements of ref. [19].

flow field does not adversely affect the LDA measurements, a fact previously shown to be the case in a round free jet. Furthermore, the presence of liquid droplets in the flow field also does not influence the response of the hot-wire type concentration probe. The probe's operating conditions are set equal to those in refs. [17–19]; therefore, according to those studies, the probe's response will again be independent of the upstream stagnation condition. Altogether, these results indicate that the laser/hot-wire technique is a reliable technique for the study of turbulent mass fluxes in complex turbulent flows with recirculation regions and fluid rotation. Even though the technique has only been demonstrated for isothermal flow, it can also be applied to non-isothermal flows without chemical reactions. In reacting flows, the technique cannot be used because the elevated temperature would affect the performance of the probe.

5. FLUX MEASUREMENTS

Since the flow is axisymmetric, as demonstrated in ref. [19], the scalar flux $\overline{wc}$ is identically zero. In order to measure $\overline{vc}$, the LDA beam volume has to lie on a vertical plane and the traverse of $\overline{vc}$ has to be along the normal to the horizontal plane passing through the tube axis. However, the difference in refractive indices between Plexiglas and the helium/air mixture and optical aberrations caused by tube wall curvature prevent the accurate measurements of $\bar{v}$. A correction lens system suggested by Durrett *et al.* [23] can be used to remedy the problem. For the present experiment, this approach, which is rather new, has not yet been tried. Consequently, only the turbulent scalar flux, $\overline{uc}$, is measured and reported in this paper. Once the laser/hot-wire technique is fully validated, the next step is to apply it to study turbulent mass fluxes in a helium/air free jet. There, the technique can be used

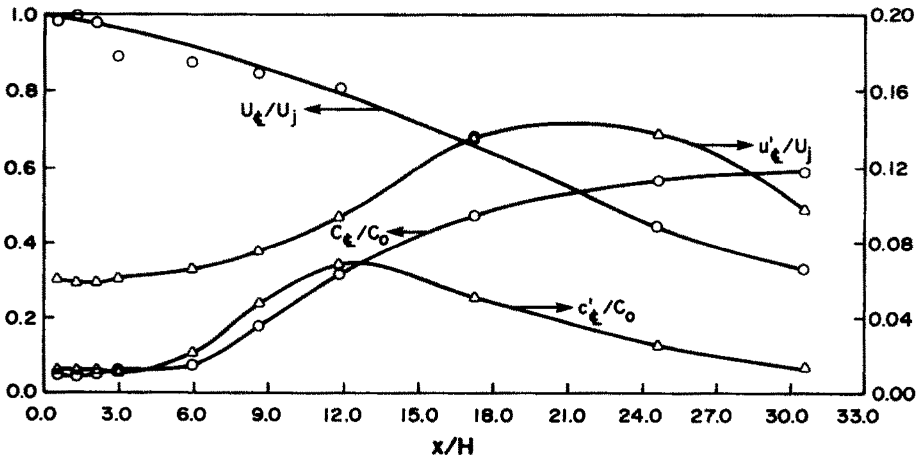


FIG. 5. Centerline distribution of axial velocity and helium volume concentration.

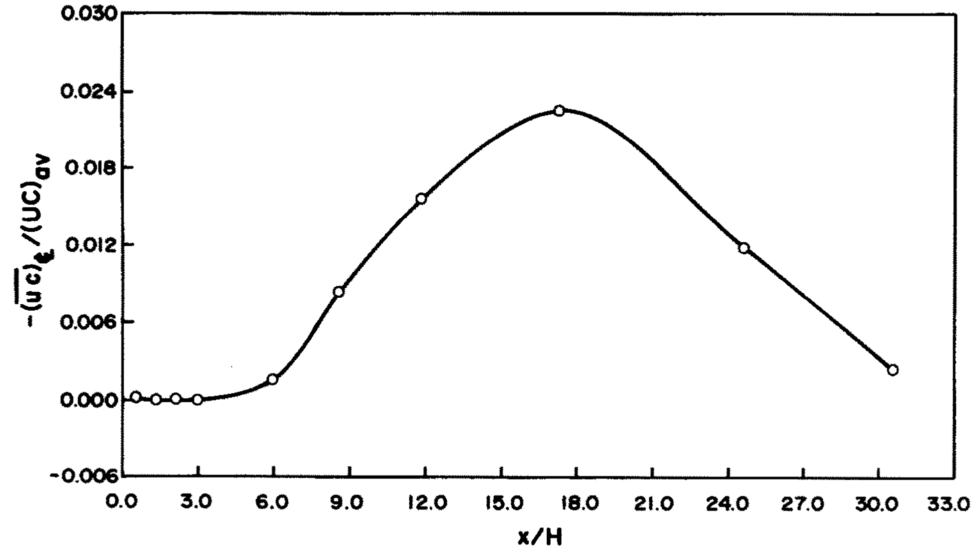


FIG. 6. Centerline distribution of $\overline{uc}$.

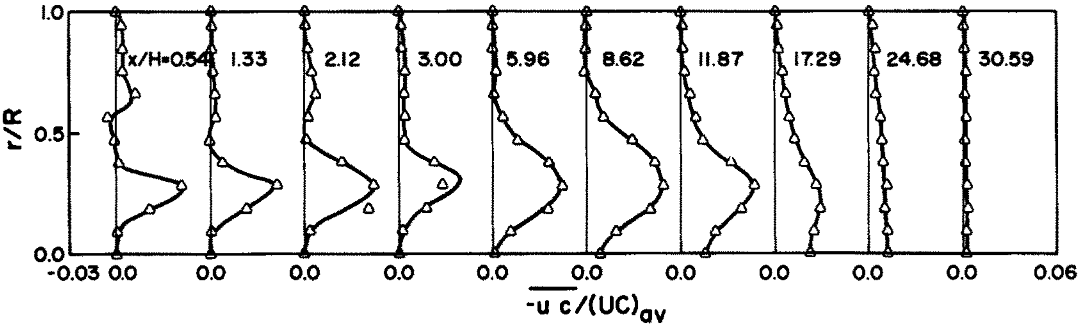


FIG. 7. Profiles of $\overline{uc}$ across the tube.

to measure $\overline{uc}$, $\overline{vc}$ and $\overline{uv}$, thus allowing the turbulent Schmidt number to be determined.

The turbulent scalar flux, $\overline{uc}$, is determined from simultaneous measurements of $\tilde{u}$ and $\tilde{c}$ and the centerline values of $\overline{uc}$ are shown in Fig. 6. As expected, $\overline{uc}$ is small compared to $(UC)_{av}$ and seldom exceeds 3% of $(UC)_{av}$. Its value is zero for $x/H \leqslant 3$, then rises to a maximum of ~2.3% of $(UC)_{av}$ at $x/H \cong 17$. Thereafter, $\overline{uc}$ decreases monotonically to zero again. The result is consistent with the U and C measurements shown in Figs. 3–5. Axial mixing between helium and air is relatively weak in the initial region downstream of the sudden expansion. This activity increases to a maximum downstream of the reattachment point and is the result of adjustment to parallel flow again. Further downstream, the flow is parallel [19] and the distributions of U and C are once again approaching uniformity. In this region, $\overline{uc}$ approaches zero just as in the initial region of the sudden-expansion flow.

The profiles of $\overline{uc}$ across the tube at ten different x locations are shown in Fig. 7. In the region $0 \leqslant x/H \leqslant 6$, there are two peaks in the profiles; one occurring at $r/R \simeq 0.29$ which remains constant along x while another occurs at $r/R \cong 0.65$ but moves towards the tube wall in the downstream direction. The first r/R location corresponds to the inner tube (Fig. 2) location, and hence, is inside the mixing layer formed between the inner air stream and the annular helium/air stream. Both the peak value of $\overline{uc}$ and its location do not change as the flow moves downstream. However, the distribution of $\overline{uc}$ spreads towards the tube wall, indicating the spread of the shear layer. The second peak in $\overline{uc}$ is very weak and occurs at approximately the location of the dividing streamline which separates the recirculating flow from the forward flow. It should be noted that $\overline{uc}$ is essentially zero in the recirculation region, $x/H < 10$. Further downstream, $\overline{uc}$ is finite near the wall, but it quickly decreases to zero in the region where the flow is uniform again. These results, therefore, show that the recirculation region is a dead region with very little mixing activity occurring between air and helium.

At very high data rates the analog output signal from the LDA frequency counter becomes nearly continuous. As indicated by the studies of Tropea [24] and Adrian and Yao [25], the use of this nearly continuous signal to calculate the velocity power spectra leads to very reliable results. Consequently, both the air and helium supply are heavily seeded so that very high

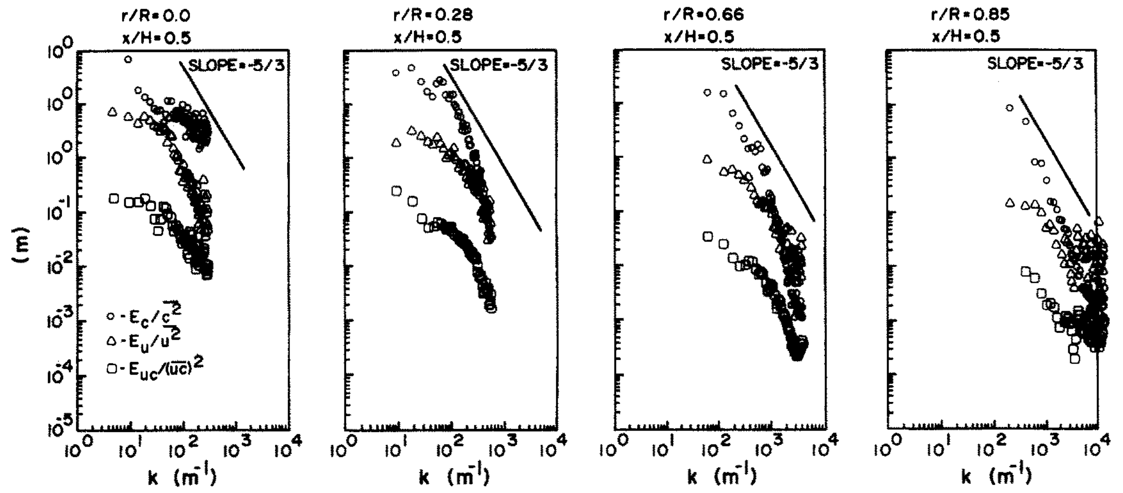


FIG. 8. Power spectra of $\overline{u^2}$, $\overline{c^2}$ and $\overline{uc}$ at $x/H = 0.5$ and four different radial locations.

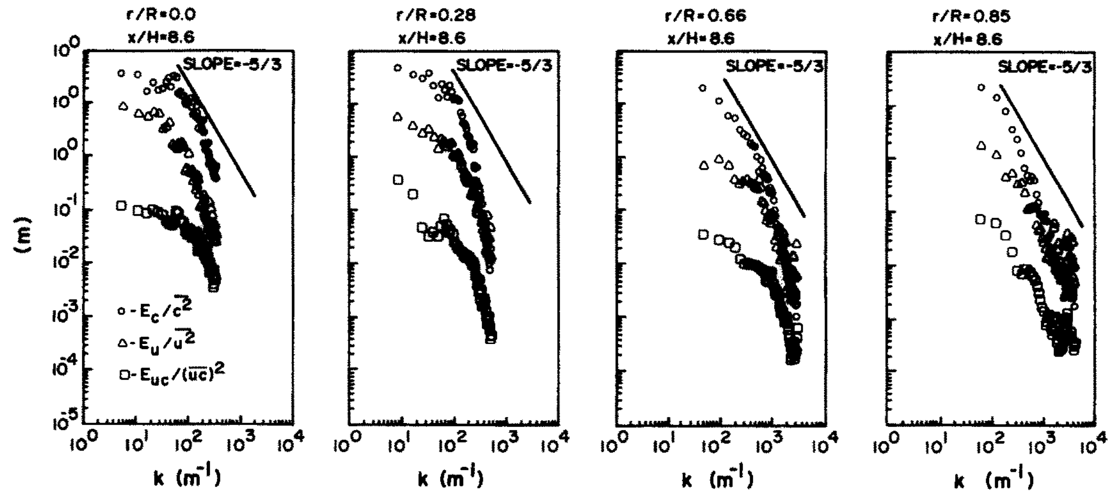


FIG. 9. Power spectra of $\overline{u^2}$, $\overline{c^2}$ and $\overline{uc}$ at $x/H = 8.6$ and four different radial locations.

data rates and nearly continuous LDA analog signals are obtained in the present study. From the time series of $\tilde{u}$ and $\tilde{c}$, the techniques of discrete Fourier transform (DFT) are used to calculate the one-dimensional power spectral densities of $\overline{u^2}$, $\overline{c^2}$ and $\overline{uc}$, denoted by E_u, E_c and E_{uc}, respectively. The DFT techniques are quite standard and are fully described in ref. [26]. Therefore, they are not repeated here. The power spectra E_u, E_c and E_{uc} are normalized by $\overline{u^2}$, $\overline{c^2}$ and $(\overline{uc})^2$, respectively, and they are presented in terms of the wave number $k = 2\pi f/U$. Only the power spectra at three different x locations are shown in Figs. 8–10. These are selected close to the sudden expansion, near the reattachment point and far downstream where the flow is nearly uniform across the tube. Along the r-direction, power spectra at four locations are shown. They are selected at $r/R = 0$, the tube centerline, $r/R = 0.28$, the location of the mixing layer, $r/R = 0.66$, the location of the separating shear layer and $r/R = 0.85$, inside the recirculation region. Therefore, the difference in behavior of these time series at different locations in the tube is clearly illustrated.

Way and Libby [4] have examined the spectra of the fluctuations of velocity and concentration in a pure helium jet. They found that the velocity and concentration spectra are quite similar and that density fluctuations have little or no effects on the high frequency portion of the spectra. This means that the spectra are in close agreement with the universal equilibrium spectra of Gibson and Schwarz [27], and suggests the existence of an inertial subrange with a decay slope of $-5/3$. Besides the data of ref. [4], which are obtained from free jet flow, there is no other data available on the spectral behavior of velocity and concentration in an internal flow. Since there is no theory which provides a basis for treating such spectra, the significance of density fluctuations on the spectra cannot be estimated *a priori*. In view of this, and the

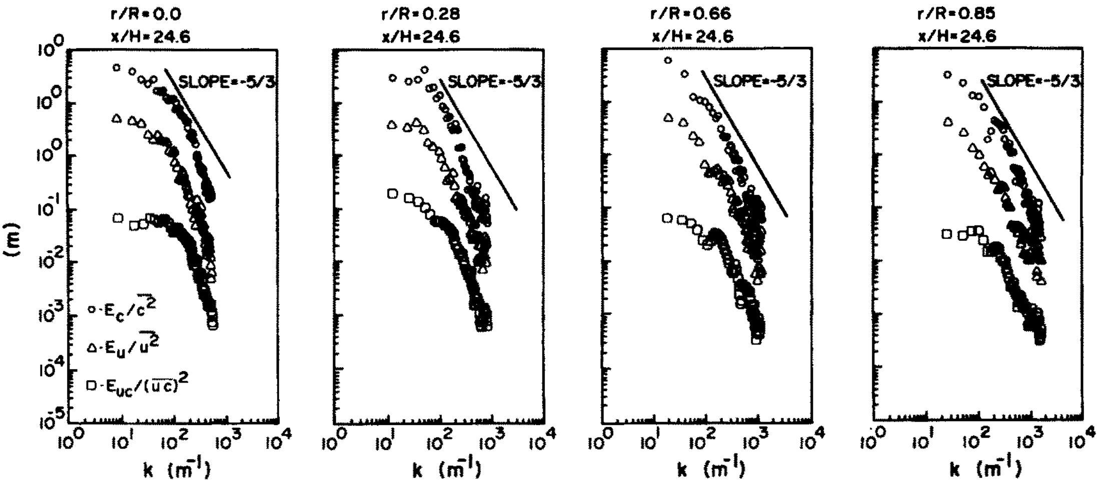


FIG. 10. Power spectra of $\overline{u^2}$, $\overline{c^2}$ and $\overline{uc}$ at $x/H = 24.6$ and four different radial locations.

lack of information on the mean rates of viscous and diffusive dissipation and turbulent Schmidt number, no attempt has been made to determine the values of the constants for the inertial subrange of the velocity and concentration spectra. Instead, the spectra are presented vs k and the inertial subrange is determined by locating a range that has a slope of $-5/3$.

The results show that in the region $0 \leqslant x/H \leqslant 3$, the spectra of $\overline{c^2}$ along the centerline do not indicate the existence of an inertial subrange (Fig. 8). This coincides with the region where $\overline{uc}$ is identically zero. However, the spectra of $\overline{u^2}$ and $\overline{uc}$ show a finite region with a slope of $-5/3$ and thus the presence of an inertial subrange (Fig. 8) and a region where the turbulence is nearly isotropic. Further downstream, beyond $x/H = 3$, an inertial subrange can be found in all velocity and concentration spectra (Figs. 9 and 10), thus indicating the isotropic behavior of the mixing phenomena and the existence of an equilibrium range in the turbulence field. Since the density fluctuations are related to the concentration fluctuations by equation (4), $\rho'/\bar{\rho}$ can be written as

$$\frac{\rho'}{\bar{\rho}} = \frac{2+\alpha}{C^{-1}+\alpha}\left(\frac{c'}{C}\right). \tag{8}$$

Figure 4 shows that the measured c'/C seldom exceeds 10% and the coefficient $(2+\alpha)/(C^{-1}+\alpha)$ is always smaller than unity, therefore, $\rho'/\bar{\rho}$ has a maximum value of 3–4% in the flow. The above results show that density fluctuations of this magnitude have little or no effect on the high frequency end of the spectra. This conclusion is in general agreement with that deduced by Way and Libby [4].

6. CONCLUSIONS

A laser/hot-wire technique has been developed and perfected for simultaneous measurements of velocity and species concentration in an isothermal, variable-density flow. Since LDA is used to measure the velocity field, reversed and swirling flows as well as flows with high levels of turbulence can be measured with accuracy. Furthermore, the hot-wire concentration probe has been demonstrated to be independent of upstream velocity variations even in a rotating field. Therefore, the laser/hot-wire technique is applicable to a wide range of flow conditions and geometry. The technique is further validated by applying it to study the variable-density flow behavior downstream of an axisymmetric sudden expansion. Since measured velocity and species concentration data on this particular flow are available, they provide a critical check on the simultaneous measurements of velocity and species concentration obtained by the LDA hot-wire technique. Excellent agreement is obtained and shows that the probe's presence does not affect the Doppler signals and the presence of small liquid droplets in the flow to serve as seeding particles for LDA measurements has little or no influence on the hot-wire response. From the measured time series of velocity and species concentration, their spectral behavior is determined and so are the correlations between velocity and species concentration and their power spectra. The power spectrum results of $\overline{u^2}$, $\overline{c^2}$ and $\overline{uc}$ show that density fluctuations in the axisymmetric sudden-expansion flow do not affect the high frequency end of the spectra. As a result, an inertial subrange with a slope of $-5/3$ is identified in these spectra.

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MESURE DE FLUX DE MASSE TURBULENT A PARTIR DE LA TECHNIQUE LASER/FIL CHAUD

Résumé—Un anémomètre laser Doppler et une sonde à concentration de type fil chaud sont utilisés pour mesurer les champs de vitesse et de concentration d'espèces simultanément dans un écoulement axisymétrique en brusque élargissement. La présence de la sonde a un effet négligeable sur les signaux Doppler. De plus la présence de petites gouttelettes liquides dans l'écoulement n'affecte pas les mesures au fil chaud. Les mesures de vitesse axiale et de concentration d'espèces sont comparées avec des données correspondantes obtenues en utilisant séparément l'anémométrie laser Doppler et la sonde à fil chaud. On obtient un excellent accord. Les flux de masse mesurés sont compatibles. La technique est utilisée pour examiner le comportement du flux de masse turbulent dans un écoulement axisymétrique avec élargissement brusque.

MESSUNGEN DES TURBULENTEN STOFFAUSTAUSCHES MIT EINER LASER/HEISSDRAHT-TECHNIK

Zusammenfassung—Ein Laser-Doppler-Anemometer und ein Heißdraht-Konzentrationsmessungs-Verfahren werden benutzt, um das Geschwindigkeits- und das Komponenten-Konzentrationsfeld gleichzeitig in einer axialsymmetrischen Expansionsströmung zu messen. Der Heißdraht hat einen vernachlässigbaren Effekt auf die Doppler-Signale. Desweiteren beeinflussen kleine Flüssigkeitstropfen in der Strömung die Heißdraht-Messungen nicht. Die Ergebnisse der axialen Geschwindigkeiten und der Komponenten-Konzentrationen wurden mit entsprechenden Daten aus jeweils separaten Messungen mit LDA und Heißdraht verglichen. Ausgezeichnete Übereinstimmung wurde erzielt. Die Messungen sind also richtig und zuverlässig. Die Technik wird benutzt, um den turbulenten Stoffaustausch in axialsymmetrischer Expansionsströmung zu untersuchen.

ИЗМЕРЕНИЯ ТУРБУЛЕНТНОГО ПОТОКА МАССЫ ЛАЗЕРНЫМ МЕТОДОМ НАГРЕТОЙ ПРОВОЛОЧКИ

Аннотация—Лазер-допплеровский анемометр и датчик концентрации по типу нагретой проволочки применялись для одновременного измерения скорости и полей концентрации компонент в осесимметричном внезапно расширяющемся потоке. Датчик не оказывал существенного влияния на показания анемометра. Более того, наличие маленьких капель жидкости в потоке не влияло на измерения, проведенные с помощью нагретой проволочки. Результаты измерений осевой скорости и концентрации, полученные лазер-допплеровским анемометром и датчиком типа нагретой проволочки в отдельности сравнивались между собой. Получено прекрасное соответствие, что свидетельствует о достоверности и надежности измерений массовых потоков. Метод применяется для изучения характеристик турбулентного потока массы в осесимметричном внезапно расширяющемся потоке.